23 total

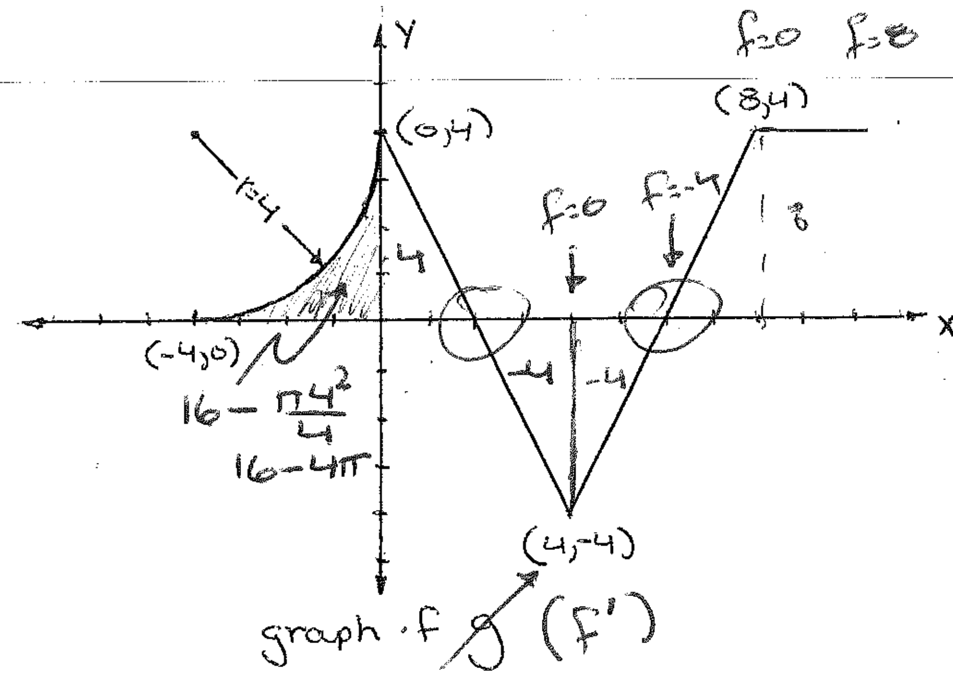
Calculus BC Exam 4 CA v.A

Integration Techniques

Dr. Wisniewski Winter 2020

Name Soln de Baybabü ε i^3 **Kaibab**

Instructions: Solve each of the following problems on separate paper showing your work so that are eligible for partial credit. A calculator is permitted on this portion of the exam. Good luck!



1. The figure above displays the graph for the function g. For $-4 \leq x \leq 10$, the function f is defined by

$$f(x) = \int_4^x g(t)\,dt. \qquad f'(x) = g(x)$$

   a. (3 Pts) Find the values of $f(6), f(-4), \& f'(0)$.

   b. (2 Pts) Identify the largest open interval(s) on which f is concave up and decreasing. Justify your answer.

   c. (2 Pts) Identify the x-coordinate(s) for which f has a local maximum. Justify your answer.

   d. (2 Pts) Find the absolute minimum value of f on $-4 \leq x \leq 10$. Justify your answer.

   e. (2 Pts) Let $h(x) = x^2 f(2x)$. Find $h'(2)$. Show your work supporting your answer.

11 pts

2. A golf ball is shot vertically into the air at 100 m/s from just over the edge of a very high vertical cliff. It is known that the velocity function for the golf ball is given by

$$v(t) = 32.662\,(4.062e^{-0.301t} - 1)$$

where $v(t)$ represents the vertical velocity of the projectile in meters/second at any time t where t is the time since the launch measured in seconds. **Make sure to include units in all your answers where appropriate.**

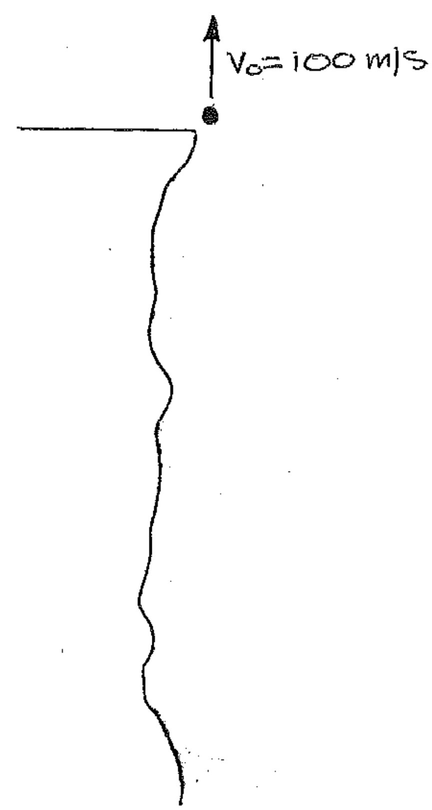
a. (1 Pt) At what time t will the golf ball reach its maximum height above its launch point?

b. (4 Pts) Use the trapezoidal rule with $n = 4$ to approximate the golf ball's vertical displacement for $0 \le t \le 8$.

c. (2 Pts) Write an integral expression that represents the actual vertical displacement of the of the ball over this same time interval. Use your calculator to evaluate the actual displacement.

d. (2 Pts) Write an expression including one or more integrals that represents the distance the ball travels for $0 \le t \le 8$. Use your calculator to evaluate the distance travelled.

e. (2 Pts) Evaluate $\lim_{t\to\infty} v(t)$. What does this physically represent?



CALC BC EXAM 4 V.A. Soln C4

$g(0) = 4$ $\frac{d}{dx}[f(2x)]$

1 a. $f(6) = \int_4^6 g(t)\,dt = \boxed{-4}$

$f(-4) = \int_4^{-4} g(t)\,dt = -\int_{-4}^{4} g(t)\,dt = -[16 - 4\pi + \cancel{4} - \cancel{4}]$

3pts

$f(-4) = \boxed{4\pi - 16}$

$f'(0) = g(0)$ FTC II $= \boxed{4}$

b. f is concave up and decreasing

f' incr   f' neg

f is concave up and decreasing on $\boxed{(4, 6)}$

2pts b/c f' (or g) is increasing which means f is concave up and b/c f' is negative which means f is decreasing.

c. f has a local max @ $x = 2$ b/c $f'(2) = 0$ and f' (or g) goes from positive to neg @ $x = 2$ which means f goes from incr to decr

2pts ∴ ∃ a local max.

d. abs min - closed-int. method

$f(-4) = 4\pi - 16 \approx -3.43$

$f(2) = 4$

$f(6) = -4$

$f(10) = 8$

abs min is $\boxed{f(6) = -4}$ 1pt

e. $h(x) = x^2 f(2x)$ 1pt

$h'(x) = 2x \cdot f(2x) + x^2 \cdot 2f'(2x)$

$h'(2) = 2(2) \cdot \cancel{f(4)} + 2^2 \cdot 2 \cdot f'(4) = 8 \cdot f'(4) = 8 \cdot g(4)$

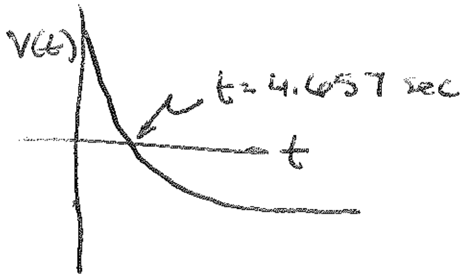
$8(-4)$

2pts

$\boxed{h'(2) = -32}$ 1pt

2 a. $\boxed{t = 4.657 \text{ sec}}$ 1pt

4.65673



b. trap w/ n=4 $0 \le t \le 8$

4pts

t	0	2	4	6	8
v(t)	100^.011^	40.0050	7.1388	-10.8625	-20.7221

$$\Delta y = \int_0^8 v(t)\,dt \approx \frac{\cancel{8}^{1}}{2\cdot 4}\Big[100^{.011} + 2(40.0050) + 2(7.1388) + 2(-10.8625) - 20.7221\Big]$$

$\boxed{\Delta y \approx 151.841 \text{ m}}$ w/100.011 → .852 m

c.

1pts

$$\Delta y = \int_0^8 v(t)\,dt = \boxed{\int_0^8 32.662\left(4.062e^{-.301t} - 1\right)dt}$$

$\boxed{\Delta y = 139.811 \text{ m}}$

d. dist traveled $= \int_0^8 \lVert v(t) \rVert\,dt$ or $\int_0^{4.657} v(t)\,dt - \int_{4.657}^{8} v(t)\,dt$

2pts

$\boxed{\text{dist traveled} = 220.518 \text{ m}}$

e. $\lim\limits_{t\to\infty} v(t) = \lim\limits_{t\to\infty} 32.662\left(4.062e^{-.301t} - 1\right)$

$= \boxed{-32.662 \text{ m/s}}$ 1pt

this represents the terminal velo!

12pts

Calculus BC Exam 4 CA v.B

Integration Techniques

Dr. Wisniewski Winter 2020

Name Solution de Bashobu **Kaibab**

Instructions: Solve each of the following problems on separate paper showing your work so that are eligible for partial credit. A calculator is permitted on this part of the exam. Good luck!

1. A golf ball is shot vertically into the air at 120 m/s from just over the edge of a very high vertical cliff. It is known that the velocity function for the golf ball is given by

$$v(t) = 32.662\,(4.674e^{-0.301t} - 1)$$

where $v(t)$ represents the vertical velocity of the projectile in meters/second at any time t where t is the time since the launch measured in seconds. **Make sure to include units in all your answers where appropriate.**

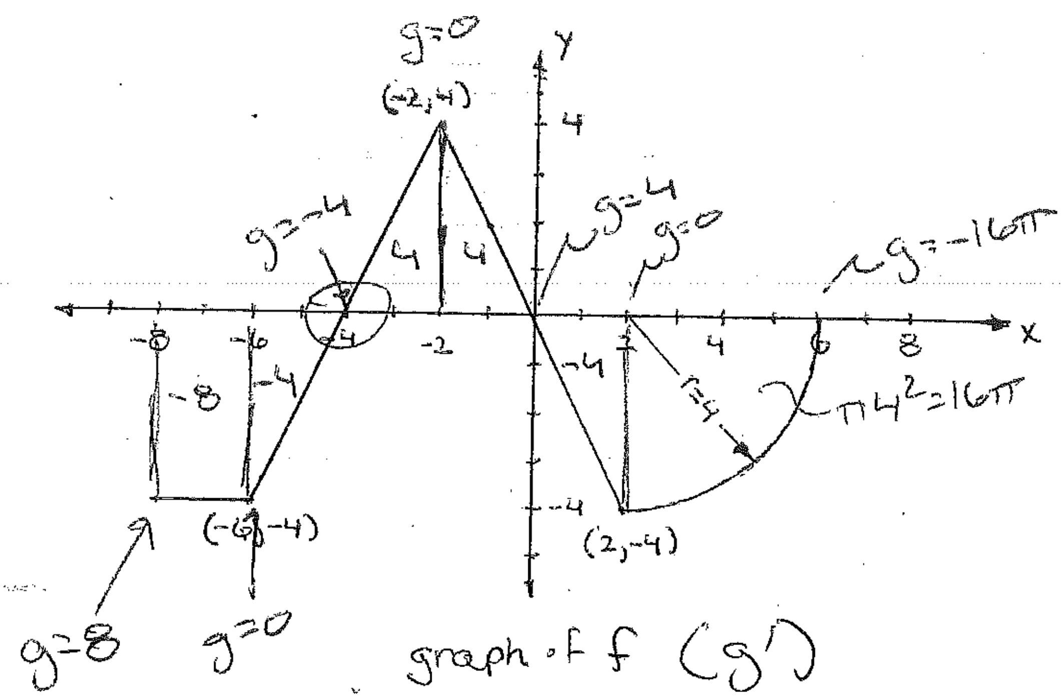
a. (1 Pt) At what time t will the golf ball reach its maximum height above its launch point?

b. (4 Pts) Use the trapezoidal rule with $n = 4$ to approximate the golf ball's vertical displacement for $0 \leq t \leq 10$.

c. (2 Pts) Write an integral expression that represents the actual vertical displacement of the of the ball over this same time interval. Use your calculator to evaluate the actual displacement.

d. (2 Pts) Write an expression including one or more integrals that represents the distance the ball travels for $0 \leq t \leq 10$. Use your calculator to evaluate the distance travelled.

e. (2 Pts) Evaluate $\lim_{t\to\infty} v(t)$. What does this physically represent?



2. The figure above displays the graph for the function f. For $-8 \le x \le 6$, the function g is defined by

$$g(x) = \int_{-2}^{x} f(t)\, dt.$$

a. (3 Pts) Find the values of $g(6), g(-8), \& \; g'(2)$.

b. (2 Pts) Identify the largest open interval(s) on which g is concave down and increasing. Justify your answer.

c. (2 Pts) Identify the x-coordinate(s) for which g has a local minimum. Justify your answer.

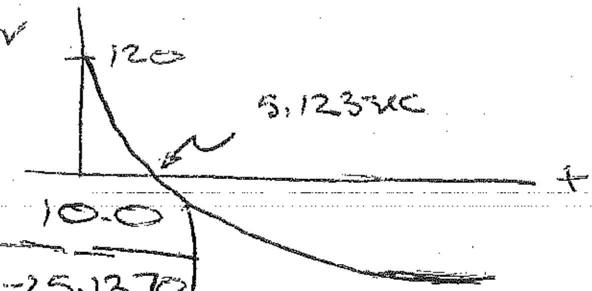
d. (2 Pts) Find the absolute maximum value of g on $-8 \le x \le 6$. Justify your answer.

e. (2 Pts) Let $J(x) = \frac{1}{4}x^2 g(x/2)$. Find $J'(-4)$. Show your work supporting your answer.

EXAM 4 VB

1. $v(t) = 32.662\,(4.674e^{-.301t} - 1)$

a. $\boxed{t = 5.123\text{ sec}}$ 1pt



b.

t	0	2.5	5.0	7.5	10.0
$v(t)$	120.000	39.2705	1.2316	-16.6918	-25.1370

$$\Delta y = \int_0^{10} v(t)\,dt \approx \frac{10}{2\cdot 4}\left[120 + 2(39.2705) + 2(1.2316) - 2(16.6918) - 25.1370\right]$$

$\boxed{\Delta y \approx 178.105\,m}$ 4pts

c. $\Delta y = \int_0^{10} v(t)\,dt = \boxed{155.563\,m}$ 4pts

D. dist traveled $= \int_0^{10} |v(t)|\,dt = \int_0^{5.123} v(t)\,dt - \int_{5.123}^{10} v(t)\,dt$

$= \boxed{307.127\,m}$ 2pts

e. $\lim_{t\to\infty} v(t) = 32.662\,m/s$ 1pt   terminal velo 1pt EC

$\boxed{12\text{pts}}$

2

a. $g(6) = \int_{-2}^{6} f(t)\,dt = \boxed{-4\pi}$

$g(-8) = \int_{-2}^{-8} f(t)\,dt = -\int_{-8}^{-2} f(t)\,dt = \boxed{8}$

3pts

1pt each

$g'(2) = f(2) = \boxed{-4}$

b. g concave down & incr $g' > 0$ and $g'' < 0$

g' decr

so $g' > 0$ and g' decr

$\therefore \boxed{(-2, 0)}$ 2pts

c. g has a local min @ $\boxed{x = -4}$ 2pts

g' going from neg to pos

g decr to incr

D.

x	g(x)
-8	8
-4	-4
0	4
6	-4π

cps

the abs max for g is $g(-8) = 8$. Closed int method.

e. $j(x) = \frac{1}{4}x^2 g(x/2)$. Find $j'(-4)$

$j'(x) = \frac{1}{2}x \cdot g(x/2) + \frac{1}{4}x^2 \cdot g'(x/2) \cdot \frac{1}{2}$

$\boxed{j'(x) = \frac{1}{2}x\,g(x/2) + \frac{1}{8}x^2 g'(x/2)}$

$j'(-4) = \cancel{\left(\tfrac{1}{2}\right)(-4)\,g(-2)}^{\,0} + \frac{1}{8}(16)\underbrace{g'(-2)}_{4}$

11pts

$\boxed{j'(-4) = 8}$